Chapter 4 Solutions

4.1. Eleven of the first 20 digits on line 109 correspond to "heads" so the proportion of heads is $\frac{11}{20} = 0.55$. This is close to 0.5, but not exactly the same because of random variation.

```
36009  19365  15412  39638
HTHHT  HTHTT  HTHHH  HTTHT
```

4.3. If you hear music (or talking) one time, you will almost certainly hear the same thing for several more checks after that. (For example, if you tune in at the beginning of a 5-minute song and check back every 5 seconds, you'll hear that same song over 30 times.)

4.4. To estimate the probability, count the number of times the dice show 7 or 11, then divide by 25. For "perfectly made" (fair) dice, the number of winning rolls will nearly always (99.4% of the time) be between 1 and 11 out of 25.

4.5. The table on the right shows information from `www.mms.com` as of this writing. The exercise specified M&M's Milk Chocolate Candies, but based on these numbers, results will be similar for other popular varieties. Of course, answers will vary, but students who take reasonably large samples should get results close to the numbers in this table. (For example, samples of size 50 will almost always be within ±12%, while size 75 should give results within ±10%.)

M&M's variety	Green %
Milk Chocolate	16%
Peanut	15%
Dark Chocolate	16%
Dark Choc. Peanut	15%
Almond	20%
Peanut Butter	20%

4.6. Out of a very large number of patients taking this medication, the fraction who experience this bad side effect is about 0.00001.

Note: *Student explanations will vary, but should make clear that 0.00001 is a long-run average rate of occurrence. Because a probability of 0.00001 is often stated as "1 in in 100,000," it is tempting to interpret this probability as meaning "exactly 1 out of every 100,000." While we* expect *about 1 occurrence of side effects out of 100,000 patients, the actual number of side effects patients is random; it might be 0, or 1, or 2, ...*

4.7. **(a)** Most answers will be between 35% and 65%. **(b)** Based on 10,000 simulated trials—more than students are expected to do—there is about an 80% chance of having a longest run of four or more (i.e., either making or missing four shots in a row), a 54% chance of getting five or more, a 31% chance of getting six or more, and a 16% chance of getting seven or more. The average ("expected") longest run length is about six.

4.8. **(a)** – **(c)** Results will vary, but after n tosses, the distribution of the proportion $\hat{p}$ is approximately Normal with mean 0.5 and standard deviation $1/(2\sqrt{n})$, while the distribution of the count of heads is approximately Normal with mean $0.5n$ and standard deviation $\sqrt{n}/2$, so using the 68–95–99.7 rule, we have the results shown in the table on the right. Note that the range for $\hat{p}$ gets narrower, while the range for the count gets wider.

n	99.7% Range for $\hat{p}$	99.7% Range for count
40	0.5 ± 0.237	20 ± 9.5
120	0.5 ± 0.137	60 ± 16.4
240	0.5 ± 0.097	120 ± 23.2
480	0.5 ± 0.068	240 ± 32.9

4.9. The true probability (assuming perfectly fair dice) is $1 - \left(\frac{5}{6}\right)^4 \doteq 0.5177$, so students should conclude that the probability is "quite close to 0.5."

4.10. The sample space is {Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday}.

4.11. One possibility: from 36 to 90 inches (largest and smallest numbers could vary but should include all possible heights).

4.12. $P(\text{Monday or Friday}) = 0.19 + 0.16 = 0.35$.

4.13. $P(\text{not Wednesday}) = 1 - 0.23 = 0.77$.

4.14. $P(\text{not } 1) = 1 - 0.301 = 0.699$.

4.15. $P(A \text{ or } B) = P(A) + P(B) = 0.301 + 0.222 = 0.523$.

4.16. For each possible value $(1, 2, \ldots, 6)$, the probability is $1/6$.

4.17. If T_k is the event "get tails on the kth flip," then T_1 and T_2 are independent, and $P(\text{two tails}) = P(T_1 \text{ and } T_2) = P(T_1)P(T_2) = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{4}$.

4.18. If A_k is the event "the kth card drawn is an ace," then A_1 and A_2 are *not* independent; in particular, if we know that A_1 occurred, then the probability of A_2 is only $\frac{3}{51}$.

4.19. There are six possible outcomes: { link1, link2, link3, link4, link5, leave }.

4.20. There are an infinite number of possible outcomes, and the description of the sample space will depend on whether the time is measured to any degree of accuracy (S is the set of all positive numbers) or rounded to (say) the nearest second ($S = \{0, 1, 2, 3, \ldots\}$), or nearest tenth of a second ($S = \{0, 0.1, 0.2, 0.3 \ldots\}$).

4.21. **(a)** The given probabilities have sum 0.55, so $P(\text{type O}) = 0.45$. **(b)** $P(\text{type O or B}) = 0.45 + 0.11 = 0.56$.

4.22. $P(\text{both are type O}) = (0.45)(0.35) = 0.1575$. $P(\text{both are the same type}) = (0.45)(0.35) + (0.40)(0.27) + (0.11)(0.26) + (0.04)(0.12) = 0.2989$.

4.23. **(a)** Not legitimate because the probabilities sum to 2. **(b)** Legitimate (for a nonstandard deck). **(c)** Legitimate (for a nonstandard die).

4.24. **(a)** The given probabilities have sum 0.41, so $P(\text{English}) = 0.59$. **(b)** $P(\text{not English}) = 1 - 0.59 = 0.41$. (Or, add the other three probabilities.)

4.25. **(a)** The given probabilities have sum 0.72, so this probability must be 0.28. **(b)** P(at least a high school education) $= 1 - P$(has not finished HS) $= 1 - 0.12 = 0.88$. (Or add the other three probabilities.)

4.26. **(a)** The given probabilities have sum 0.81, so P(other topic) $= 0.19$. **(b)** P(adult or scam) $= 0.145 + 0.142 = 0.287$.

Note: *An underlying assumption here is that each piece of spam falls into exactly one category.*

4.27. The probabilities of 2, 3, 4, and 5 are unchanged (1/6), so P(⚀ or ⚁) must still be 1/3. If P(⚁) $= 0.2$, then P(⚀) $= \frac{1}{3} - 0.2 = 0.1\overline{3}$ (or $\frac{2}{15}$).

Face	⚀	⚁	⚂	⚃	⚄	⚅
Probability	$0.1\overline{3}$	1/6	1/6	1/6	1/6	0.2

4.28. **(a)** It is legitimate because every person must fall into exactly one category, and the probabilities add up to 1. **(b)** $P(A) = 0.125 = 0.000 + 0.003 + 0.060 + 0.062$. **(c)** B^c is the event "the person chosen is not white." $P(B^c) = 1 - P(B) = 1 - (0.060 + 0.691) = 0.249$. **(d)** $P(A^c \text{ and } B) = 0.691$ is the probability that a randomly chosen American is a non-Hispanic white.

4.29. For example, the probability for O-positive blood is $(0.45)(0.84) = 0.378$ and for O-negative $(0.45)(0.16) = 0.072$.

Blood type	O+	O–	A+	A–	B+	B–	AB+	AB–
Probability	0.3780	0.0720	0.3360	0.0640	0.0924	0.0176	0.0336	0.0064

4.30. We found in Exercise 4.28 that $P(A) = 0.125$ and that $P(B) = 0.751$. (We actually computed $P(B^c) = 0.249$.) Because $P(A)P(B) \doteq 0.094$ is not equal to $P(A \text{ and } B) = 0.060$ (from the table in Exercise 4.22), A and B are *not* independent.

4.31. **(a)** All are equally likely; the probability is 1/38. **(b)** Because 18 slots are red, the probability of a red is $P(\text{red}) = \frac{18}{38} \doteq 0.474$. **(c)** There are 12 winning slots, so P(win a column bet) $= \frac{12}{38} \doteq 0.316$.

4.32. **(a)** There are six arrangements of the digits 4, 5, and 6 (456, 465, 546, 564, 645, 654), so that $P(\text{win}) = \frac{6}{1000} = 0.006$. **(b)** With the digits 2, 1, and 2, there are only three distinct arrangements (212, 221, 122), so $P(\text{win}) = \frac{3}{1000} = 0.003$.

4.33. **(a)** There are $10^4 = 10{,}000$ possible PINs (0000 through 9999).* **(b)** The probability that a PIN has *no* 0s is 0.9^4 (because there are 9^4 PINs that can be made from the nine nonzero digits), so the probability of at least one 0 is $1 - 0.9^4 = 0.3439$.

*If we assume that PINs cannot have leading 0s, then there are only 9000 possible codes (1000–9999), and the probability of at least one 0 is $1 - \frac{9^4}{9000} = 0.271$.

4.34. $P(\text{none are O-negative}) = (1 - 0.07)^{10} \doteq 0.4840$, so $P(\text{at least one is O-negative}) \doteq 1 - 0.4840 = 0.5160$.

4.35. If we assume that each site is independent of the others (and that they can be considered as a random sample from the collection of sites referenced in scientific journals), then $P(\text{all seven are still good}) = 0.87^7 \doteq 0.3773$.

4.36. (a) About 0.33: $P(\text{no calls reach a live person}) \doteq (1 - 0.2)^5 = 0.8^5 = 0.32768$.
(b) About 0.66: $P(\text{no NY calls reach a live person}) \doteq 0.92^5 \doteq 0.65908$.

4.37. This computation would only be correct if the events "a randomly selected person is at least 75" and "a randomly selected person is a woman" were independent. This is likely not true; in particular, as women have a greater life expectancy than men, this fraction is probably greater than 3%.

4.38. As $P(\text{R}) = \frac{2}{6}$ and $P(\text{G}) = \frac{4}{6}$, and successive rolls are independent, the respective probabilities are:
$\left(\frac{2}{6}\right)^4\left(\frac{4}{6}\right) = \frac{2}{243} \doteq 0.00823$, $\left(\frac{2}{6}\right)^4\left(\frac{4}{6}\right)^2 = \frac{4}{729} \doteq 0.00549$, and $\left(\frac{2}{6}\right)^5\left(\frac{4}{6}\right) = \frac{2}{729} \doteq 0.00274$

4.39. (a) $(0.65)^3 \doteq 0.2746$ (under the random walk theory). **(b)** 0.35 (because performance in separate years is independent). **(c)** $(0.65)^2 + (0.35)^2 = 0.545$.

4.40. For any event A, along with its complement A^c, we have $P(S) = P(A \text{ or } A^c)$ because "A or A^c" includes all possible outcomes (that is, it is the entire sample space S). By Rule 2, $P(S) = 1$, and by Rule 3, $P(A \text{ or } A^c) = P(A) + P(A^c)$, because A and A^c are disjoint. Therefore, $P(A) + P(A^c) = 1$, from which Rule 4 follows.

4.41. Note that $A = (A \text{ and } B) \text{ or } (A \text{ and } B^c)$, and the events $(A \text{ and } B)$ and $(A \text{ and } B^c)$ are disjoint, so Rule 3 says that $P(A) = P\big((A \text{ and } B) \text{ or } (A \text{ and } B^c)\big) = P(A \text{ and } B) + P(A \text{ and } B^c)$. If $P(A \text{ and } B) = P(A)P(B)$, then we have $P(A \text{ and } B^c) = P(A) - P(A)P(B) = P(A)(1 - P(B))$, which equals $P(A)P(B^c)$ by the complement rule.

4.42. (a) Hannah and Jacob's children can have alleles AA, BB, or AB, so they can have blood type A, B, or AB. (The table on the right shows the possible combinations.) **(b)** Either note that the four combinations in the table are equally likely, or compute:

	A	B
A	AA	AB
B	AB	BB

$$P(\text{type A}) = P(\text{A from Hannah and A from Jacob}) = P(\text{A}_H)P(\text{A}_J) = 0.5^2 = 0.25$$
$$P(\text{type B}) = P(\text{B from Hannah and B from Jacob}) = P(\text{B}_H)P(\text{B}_J) = 0.5^2 = 0.25$$
$$P(\text{type AB}) = P(\text{A}_H)P(\text{B}_J) + P(\text{B}_H)P(\text{A}_J) = 2 \cdot 0.25 = 0.5$$

4.43. (a) Nancy and David's children can have alleles BB, BO, or OO, so they can have blood type B or O. (The table on the right shows the possible combinations.) **(b)** Either note that the four combinations in the table are equally likely or compute $P(\text{type O}) = P(\text{O from Nancy and O from David}) = 0.5^2 = 0.25$ and $P(\text{type B}) = 1 - P(\text{type O}) = 0.75$.

	B	O
B	BB	BO
O	BO	OO

4.44. Any child of Jennifer and José has a 50% chance of being type A (alleles AA or AO), and each child inherits alleles independently of other children, so $P(\text{both are type A}) = 0.5^2 = 0.25$. For one child, we have $P(\text{type A}) = 0.5$ and $P(\text{type AB}) = P(\text{type B}) = 0.25$, so that $P(\text{both have same type}) = 0.5^2 + 0.25^2 + 0.25^2 = 0.375 = \frac{3}{8}$.

	A	O
A	AA	AO
B	AB	BO

4.45. **(a)** Any child of Jasmine and Joshua has an equal (1/4) chance of having blood type AB, A, B, or O (see the allele combinations in the table). Therefore, $P(\text{type O}) = 0.25$. **(b)** $P(\text{all three have type O}) = 0.25^3 = 0.015625 = \frac{1}{64}$. $P(\text{first has type O, next two do not}) = 0.25 \cdot 0.75^2 = 0.140625 = \frac{9}{64}$.

	A	O
B	AB	BO
O	AO	OO

4.46. $P(\text{D or F}) = P(X = 0 \text{ or } X = 1) = 0.05 + 0.04 = 0.09$.

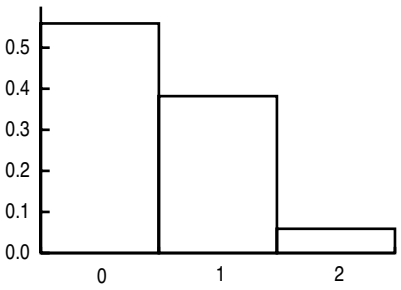
4.47. If H is the number of heads, then the distribution of H is as given on the right. $P(H = 0)$, the probability of two tails was previously computed in Exercise 4.17.

Value of H	0	1	2
Probabilities	1/4	1/2	1/4

4.48. $P(0.1 < X < 0.4) = 0.3$.

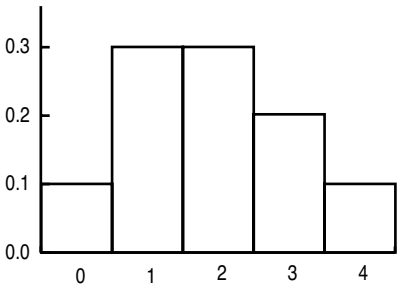
4.49. **(a)** See also the solution to Exercise 4.20. If we view this time as being measured to any degree of accuracy, it is continuous; if it is rounded, it is discrete. **(b)** A count like this must be a whole number, so it is discrete. **(c)** Incomes—whether given in dollars and cents, or rounded to the nearest dollar—are discrete.

4.50. **(a)** $0.559 + 0.382 + 0.059 = 1$. **(b)** Histogram on the right. **(c)** $P(\text{at least one ace}) = 0.441$, which can be computed either as $0.382 + 0.059$ or $1 - 0.559$.

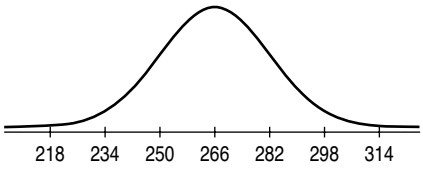


4.51. **(a)** Histogram on the right. **(b)** "At least one nonword error" is the event "$X \geq 1$" (or "$X > 0$"). $P(X \geq 1) = 1 - P(X = 0) = 0.9$. **(c)** "$X \leq 2$" is "no more than two nonword errors," or "fewer than three nonword errors."

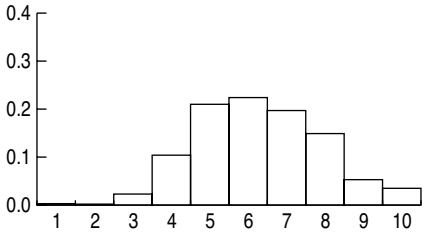
$$P(X \leq 2) = 0.7 = P(X = 0) + P(X = 1) + P(X = 2)$$
$$= 0.1 + 0.3 + 0.3$$
$$P(X < 2) = 0.4 = P(X = 0) + P(X = 1) = 0.1 + 0.3$$

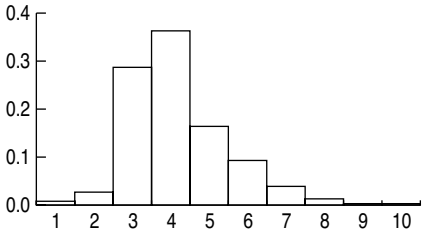


4.52. **(a)** Curve on the right. A good procedure is to draw the curve first, locate the points where the curvature changes, then mark the horizontal axis. Students may at first make mistakes like drawing a half-circle instead of the correct "bell-shaped" curve or being careless about locating the standard deviation. **(b)** About 0.017: $P(Y > 300) = P\left(\frac{Y-266}{16} > \frac{300-266}{16}\right) = P(Z > 2.125)$. Software gives 0.0168; Table A gives 0.0166 for -2.13 and 0.0170 for -2.12 (so the average is again 0.0168).

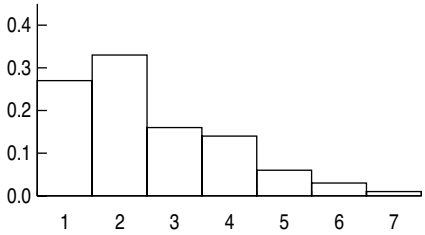


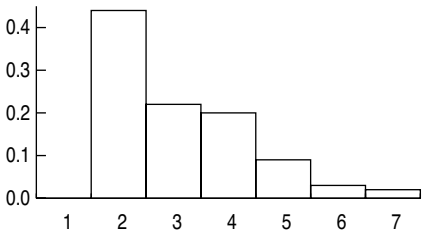
4.53. The two histograms are shown below. Rented housing typically has fewer rooms, and its distribution is considerably more skewed than the owned-housing distribution.





4.54. The two histograms are shown below. The most obvious difference is that a "family" must have at least two people. Otherwise, the family-size distribution has slightly larger probabilities for 2, 3, or 4, while for large family/household sizes, the differences between the distributions are small.

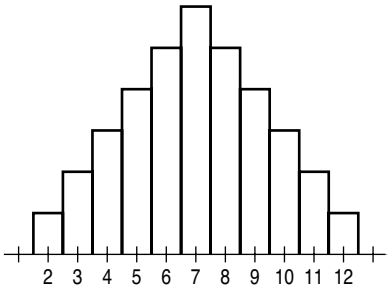




4.55. **(a)** "The unit has six or more rooms" is "$X \geq 6$" or "$X > 5$." $P(X \geq 6) = 0.224 + 0.197 + 0.149 + 0.053 + 0.035 = 0.658$. **(b)** "$X > 6$" is "the unit has more than six (or at least seven) rooms." $P(X > 6) = 0.434$. **(c)** With discrete distributions, one must pay attention to whether or not the endpoints should be included in the probability computation (i.e., pay attention to whether you have "greater than" or "greater than or equal to").

4.56. **(a)** The pairs are given on the next page. We must assume that we can distinguish between, for example, "(1,2)" and "(2,1)"; otherwise, the outcomes are not equally likely. **(b)** Each pair has probability 1/36. **(c)** The value of X is given below each pair. For the distribution (given below), we see (for example) that there are four pairs that add to 5, so $P(X = 5) = \frac{4}{36}$. Histogram below, right. **(d)** $P(7 \text{ or } 11) = \frac{6}{36} + \frac{2}{36} = \frac{8}{36} = \frac{2}{9}$. **(e)** $P(\text{not } 7) = 1 - \frac{6}{36} = \frac{5}{6}$.

(1,1) 2	(1,2) 3	(1,3) 4	(1,4) 5	(1,5) 6	(1,6) 7
(2,1) 3	(2,2) 4	(2,3) 5	(2,4) 6	(2,5) 7	(2,6) 8
(3,1) 4	(3,2) 5	(3,3) 6	(3,4) 7	(3,5) 8	(3,6) 9
(4,1) 5	(4,2) 6	(4,3) 7	(4,4) 8	(4,5) 9	(4,6) 10
(5,1) 6	(5,2) 7	(5,3) 8	(5,4) 9	(5,5) 10	(5,6) 11
(6,1) 7	(6,2) 8	(6,3) 9	(6,4) 10	(6,5) 11	(6,6) 12



Value of X	2	3	4	5	6	7	8	9	10	11	12
Probability	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

4.57. The possible values of Y are 1, 2, 3, . . . ,12, each with probability 1/12. Aside from drawing a diagram showing all the possible combinations, one can reason that the first (regular) die is equally likely to show any number from 1 through 6. Half of the time, the second roll shows 0, and the rest of the time it shows 6. Each possible outcome therefore has probability $\frac{1}{6} \cdot \frac{1}{2}$.

4.58. The table on the right shows the additional columns to add to the table given in the solution to Exercise 4.48. There are 48 possible (equally-likely) combinations.

Value of X	2	3	4	5	6	7	8	9	10	11	12	13	14
Probability	$\frac{1}{48}$	$\frac{2}{48}$	$\frac{3}{48}$	$\frac{4}{48}$	$\frac{5}{48}$	$\frac{6}{48}$	$\frac{6}{48}$	$\frac{6}{48}$	$\frac{5}{48}$	$\frac{4}{48}$	$\frac{3}{48}$	$\frac{2}{48}$	$\frac{1}{48}$

(1,7) 8	(1,8) 9
(2,7) 9	(2,8) 10
(3,7) 10	(3,8) 11
(4,7) 11	(4,8) 12
(5,7) 12	(5,8) 13
(6,7) 13	(6,8) 14

4.59. **(a)** W can be 0, 1, 2, or 3. **(b)** See the top two lines of the table below. **(c)** The distribution is given in the bottom two lines of the table. For example, $P(W = 0) = (0.73)(0.73)(0.73) \doteq 0.3890$, and in the same way, $P(W = 3) = 0.27^3 \doteq 0.1597$. For $P(W = 1)$, note that each of the three arrangements that give $W = 1$ have probability $(0.73)(0.73)(0.27) = 0.143883$, so $P(W = 1) = 3(0.143883) \doteq 0.4316$. Similarly, $P(W = 2) = 3(0.73)(0.27)(0.27) \doteq 0.1597$.

Arrangement	DDD	DDF	DFD	FDD	FFD	FDF	DFF	FFF
Probability	0.3890	0.1439	0.1439	0.1439	0.0532	0.0532	0.0532	0.0197
Value of W	0	1			2			3
Probability	0.3890	0.4316			0.1597			0.0197

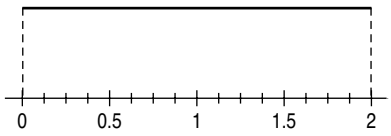
4.60. Let "S" mean that a student supports funding and "O" mean that the student opposes funding. **(a)** $P(\text{SSO}) = (0.6)(0.6)(0.4) = 0.144$. **(b)** See the top two lines of the table below. **(c)** The distribution is given in the bottom two lines of the table. For example, $P(X = 0) = (0.6)(0.6)(0.6) = 0.216$, and in the same way, $P(X = 3) = 0.4^3 = 0.064$. For $P(X = 1)$, note that each of the three arrangements that give $X = 1$ have probability 0.144, so $P(X = 1) = 3(0.144) = 0.432$. Similarly, $P(X = 2) = 3(0.6)(0.4)(0.4) = 0.288$. **(d)** A majority means $X \geq 2$; $P(X \geq 2) = 0.288 + 0.064 = 0.352$.

Arrangement	SSS	SSO	SOS	OSS	OOS	OSO	SOO	OOO
Probability	0.216	0.144	0.144	0.144	0.096	0.096	0.096	0.064
Value of X	0	1			2			3
Probability	0.216	0.432			0.288			0.064

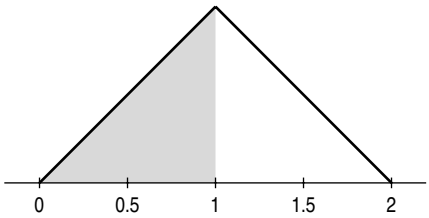
4.61. **(a)** $P(X < 0.4) = 0.4$. **(b)** $P(X \leq 0.4) = 0.4$. **(c)** For continuous random variables, "equal to" has no effect on the probability; that is, $P(X = c) = 0$ for any value of c.

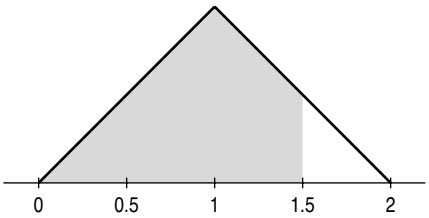
4.62. **(a)** $P(X \geq 0.35) = 0.65$. **(b)** $P(X = 0.35) = 0$. **(c)** $P(0.35 < X < 1.35) = P(0.35 < X < 1) = 0.65$. **(d)** $P(0.15 \leq X \leq 0.25 \text{ or } 0.8 \leq X \leq 0.9) = 0.1 + 0.1 = 0.2$. **(e)** $P(\text{not } [0.3 \leq X \leq 0.7]) = 1 - P(0.3 \leq X \leq 0.7) = 1 - 0.4 = 0.6$.

4.63. **(a)** The height should be $\frac{1}{2}$ since the area under the curve must be 1. The density curve is at the right. **(b)** $P(Y \leq 1.5) = \frac{1.5}{2} = 0.75$. **(c)** $P(0.6 < Y < 1.7) = \frac{1.1}{2} = 0.55$. **(d)** $P(Y \geq 0.9) = \frac{1.1}{2} = 0.55$.



4.64. **(a)** The area of a triangle is $\frac{1}{2}bh = \frac{1}{2}(2)(1) = 1$. **(b)** $P(Y < 1) = 0.5$. **(c)** $P(Y < 1.5) = 0.875$; the easiest way to compute this is to note that the unshaded area is a triangle with area $\frac{1}{2}(0.5)(0.5) = 0.125$.





4.65. $P(8 \leq \bar{x} \leq 10) = P\left(\frac{8-9}{0.075} \leq \frac{\bar{x}-9}{0.075} \leq \frac{10-9}{0.075}\right) = P(-13.3 \leq Z \leq 13.3)$. This probability is essentially 1; $\bar{x}$ will almost certainly estimate μ within ± 1 (in fact, it will almost certainly be much closer than this).

4.66. **(a)** $P(0.52 \leq \hat{p} \leq 0.60) = P\left(\frac{0.52-0.56}{0.019} \leq \frac{\hat{p}-0.56}{0.019} \leq \frac{0.60-0.56}{0.019}\right) = P(-2.11 \leq Z \leq 2.11) = 0.9826 - 0.0174 = 0.9652$. **(b)** $P(\hat{p} \geq 0.72) = P\left(\frac{\hat{p}-0.56}{0.019} \geq \frac{0.72-0.56}{0.019}\right) = P(Z \geq 8.42)$; this is basically 0.

4.67. The possible values of X are \$0 and \$1, each with probability 0.5 (because the coin is fair). The mean is $\$0\left(\frac{1}{2}\right) + \$1\left(\frac{1}{2}\right) = \$0.50$.

4.69. If $Y = 15 + 8X$, then $\mu_Y = 15 + 8\mu_X = 15 + 8(10) = 95$.

4.70. If $W = 0.5U + 0.5V$, then $\mu_W = 0.5\mu_U + 0.5\mu_V = 0.5(20) + 0.5(20) = 20$.

4.71. First we note that $\mu_X = 0(0.5) + 2(0.5) = 1$, so $\sigma_X^2 = (0-1)^2(0.5) + (2-1)^2(0.5) = 1$ and $\sigma_X = \sqrt{\sigma_X^2} = 1$.

4.72. The mean number of aces is $\mu_X = (0)(0.559) + (1)(0.382) + (2)(0.059) = 0.5$.

4.73. The average grade is $\mu = (0)(0.05) + (1)(0.04) + (2)(0.20) + (3)(0.40) + (4)(0.31) = 2.88$.

4.74. The mean number of nonword errors is $(0)(0.1)+(1)(0.3)+(2)(0.3)+(3)(0.2)+(4)(0.1) = 1.9$, and the mean number of word errors is $(0)(0.4) + (1)(0.3) + (2)(0.2) + (3)(0.1) = 1$.

4.75. For owner-occupied units, the mean is:

$$(1)(0.003) + (2)(0.002) + (3)(0.023) + (4)(0.104) + (5)(0.210)+$$
$$(6)(0.224) + (7)(0.197) + (8)(0.149) + (9)(0.053) + (10)(0.035) = 6.284 \text{ rooms}$$

For rented units, the mean is:

$$(1)(0.008) + (2)(0.027) + (3)(0.287) + (4)(0.363) + (5)(0.164)+$$
$$(6)(0.093) + (7)(0.039) + (8)(0.013) + (9)(0.003) + (10)(0.003) = 4.187 \text{ rooms}$$

This agrees with the observation in Exercise 4.53 that rented housing typically has fewer rooms.

4.76. **(a)** The mean of Y is $\mu_Y = 1$—the obvious balance point of the triangle. **(b)** Both X_1 and X_2 have mean $\mu_{X_1} = \mu_{X_2} = 0.5$ and $\mu_Y = \mu_{X_1} + \mu_{X_2}$.

4.77. The owned-unit distribution seems to be more spread out, and the two standard deviations confirm this impression: $\sigma_o \doteq 1.6399$ and $\sigma_r \doteq 1.3077$ rooms. The full details of these computations are not shown, but for example, the variance of the number of rooms in an owner-occupied unit looks like this:

$$\sigma_o^2 = (1-\mu_o)^2(0.003) + (2-\mu_o)^2(0.008) + \cdots + (10-\mu_o)^2(0.035) \doteq 2.6893$$

In the same way, $\sigma_r^2 \doteq 1.7100$. Taking square roots completes the task.

4.78. Let N and W be nonword and word error counts. In Exercise 4.74, we found $\mu_N = 1.9$ errors and $\mu_W = 1$ error. The variances of these distributions are $\sigma_N^2 = 1.29$ and $\sigma_W^2 = 1$, so the standard deviations are $\sigma_N \doteq 1.1358$ and $\sigma_W = 1$ errors. The mean total error count is $\mu_N + \mu_W = 2.9$ errors for both cases. **(a)** If error counts are independent, $\sigma_{N+W}^2 = \sigma_N^2 + \sigma_W^2 = 2.29$ and $\sigma_{N+W} \doteq 1.5133$ errors. (Note we add the *variances,* not the standard deviations.) **(b)** With $\rho = 0.4$, $\sigma_{N+W}^2 = \sigma_N^2 + \sigma_W^2 + 2\rho\sigma_N\sigma_W \doteq 2.29 + 0.9086 = 3.1986$ and $\sigma_{N+W} \doteq 1.7885$ errors.

4.79. **(a)** The mean for one coin is $\mu_1 = (0)\left(\frac{1}{2}\right) + (1)\left(\frac{1}{2}\right) = 0.5$ and the variance is $\sigma_1^2 = (0 - 0.5)^2\left(\frac{1}{2}\right) + (1 - 0.5)^2\left(\frac{1}{2}\right) = 0.25$, so the standard deviation is $\sigma_1 = 0.5$. **(b)** Multiply μ_1 and σ_1^2 by 4: $\mu_4 = 4\mu_1 = 2$ and $\sigma_4^2 = 4\sigma_1^4 = 1$, so $\sigma_4 = 1$. **(c)** The computations (not shown here) are more tedious, but the results are the same. Note that because of the symmetry of the distribution, we do not need to compute the mean to see that $\mu_4 = 2$; this is the obvious balance point of the probability histogram in Figure 4.7.

4.80. If S and E are the results from the six- and eight-sided dice (respectively), then $\mu_S = 3.5$ and $\mu_E = 4.5$ by symmetry (this can be confirmed by computation). Therefore, $\mu_{S+E} = 8$.

For the variances:

$$\sigma_S^2 = (1 - 3.5)^2\left(\tfrac{1}{6}\right) + (2 - 3.5)^2\left(\tfrac{1}{6}\right) + (3 - 3.5)^2\left(\tfrac{1}{6}\right) + (4 - 3.5)^2\left(\tfrac{1}{6}\right) + (5 - 3.5)^2\left(\tfrac{1}{6}\right) + (6 - 3.5)^2\left(\tfrac{1}{6}\right) = \tfrac{35}{12} \doteq 2.9167$$

By a similar computation, $\sigma_E^2 = 5.25$. Therefore, $\sigma_{S+E}^2 = 8\frac{1}{6} \doteq 8.1667$, and the standard deviation is $\sigma_{S+E} \doteq 2.8577$.

4.81. With R as the rod length and B_1 and B_2 the bearing lengths, we have $\mu_{B_1+R+B_2} = 12 + 2 \cdot 2 = 16$ cm and $\sigma_{B_1+R+B_2} = \sqrt{0.004^2 + 2 \cdot 0.001^2} \doteq 0.004243$ cm.

4.82. **(a)** $d_1 = 2\sigma_R = 0.008$ cm and $d_2 = 2\sigma_B = 0.002$ cm. **(b)** The natural tolerance of the assembled parts is $2\sigma_{B_1+R+B_2} \doteq 0.008485$ cm.

4.83. **(a)** Not independent: Knowing the total X of the first two cards tells us something about the total Y for three cards. **(b)** Independent: Separate rolls of the dice should be independent.

4.84. Divide the given values by 2.54: $\mu \doteq 69.6063$ in and $\sigma \doteq 2.8346$ in.

4.85. With $\rho = 1$, we have:

$$\sigma_{X+Y}^2 = \sigma_X^2 + \sigma_Y^2 + 2\rho\sigma_X\sigma_Y = \sigma_X^2 + \sigma_Y^2 + 2\sigma_X\sigma_Y = (\sigma_X + \sigma_Y)^2$$

And of course, $\sigma_{X+Y} = \sqrt{(\sigma_X + \sigma_Y)^2} = \sigma_X + \sigma_Y$.

4.86. The mean of X is $(\mu - \sigma)(0.5) + (\mu + \sigma)(0.5) = \mu$, and the standard deviation is $\sqrt{(\mu - \sigma - \mu)^2(0.5) + (\mu + \sigma - \mu)^2(0.5)} = \sqrt{\sigma^2} = \sigma$.

4.87. Although the probability of having to pay for a total loss for one or more of the 10 policies is very small, if this were to happen, it would be financially disastrous. On the other hand, for thousands of policies, the law of large numbers says that the average claim on many policies will be close to the mean, so the insurance company can be assured that the premiums they collect will (almost certainly) cover the claims.

4.88. The total loss T for 10 fires has mean $\mu_T = 10 \cdot \$300 = \$3000$, and standard deviation $\sigma_T = \sqrt{10 \cdot \$400^2} = \$400\sqrt{10} \doteq \$1264.91$. The average loss is $T/10$, so $\mu_{T/10} = \frac{1}{10}\mu_T = \$300$ and $\sigma_{T/10} = \frac{1}{10}\sigma_T \doteq \$126.49$.

The total loss T for 12 fires has mean $\mu_T = 12 \cdot \$300 = \$3600$, and standard deviation $\sigma_T = \sqrt{12 \cdot \$400^2} = \$400\sqrt{12} \doteq \$1385.64$. The average loss is $T/12$, so $\mu_{T/12} = \frac{1}{12}\mu_T = \$300$ and $\sigma_{T/12} = \frac{1}{12}\sigma_T = \$115.47$.

Note: *The mean of the average loss is the same regardless of the number of policies, but the standard deviation decreases as the number of policies increases. With thousands of policies, the standard deviation is very small, so the average claim will be close to \$300, as was stated in the solution to the previous problem.*

4.89. **(a)** Add up the given probabilities and subtract from 1; this gives $P(\text{man does not die in the next five years}) = 0.99749$. **(b)** The distribution of income (or loss) is given below. Multiplying each possible value by its probability gives the mean intake $\mu \doteq \$623.22$.

Age at death	21	22	23	24	25	Survives
Loss or income	−\$99,825	−\$99,650	−\$99,475	−\$99,300	−\$99,125	\$875
Probability	0.00039	0.00044	0.00051	0.00057	0.00060	0.99749

4.90. The mean μ of the company's "winnings" (premiums) and their "losses" (insurance claims) is positive. Even though the company will lose a large amount of money on a small number of policyholders who die, it will gain a small amount on the majority. The law of large numbers says that the average "winnings" minus "losses" should be close to μ, and overall the company will almost certainly show a profit.

4.91. With $R = 0.8W + 0.2Y$, we have $\mu_R = 0.8\mu_W + 0.2\mu_Y = 11.116\%$ and:

$$\sigma_R = \sqrt{(0.8\sigma_W)^2 + (0.2\sigma_Y)^2 + 2\rho_{WY}(0.8\sigma_W)(0.2\sigma_Y)} \doteq 15.9291\%$$

4.92. With $\rho_{WY} = 0$, the standard deviation drops to $\sqrt{(0.8\sigma_W)^2 + (0.2\sigma_Y)^2} \doteq 14.3131\%$. The mean is unaffected by the correlation.

4.93. With $R = 0.6W + 0.2X + 0.2Y$, we have $\mu_R = 0.6\mu_W + 0.2\mu_X + 0.2\mu_Y = 10.184\%$ and:

$$\begin{aligned}\sigma_R^2 &= (0.6\sigma_W)^2 + (0.2\sigma_X)^2 + (0.2\sigma_Y)^2 \\ &\quad + 2\rho_{WX}(0.6\sigma_W)(0.2\sigma_X) + 2\rho_{WY}(0.6\sigma_W)(0.2\sigma_Y) + 2\rho_{XY}(0.2\sigma_X)(0.2\sigma_Y) \\ &\doteq 152.3788\end{aligned}$$

so that $\sigma_R \doteq 12.3442\%$. The benefit of this diversification is reduced variability (σ_R is smaller than either σ_W and σ_Y), but the mean return is reduced because of the inclusion of the Bond Fund.

4.94. The events "roll a 3" and "roll a 5" are disjoint, so $P(3 \text{ or } 5) = P(3) + P(5) = \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$.

4.95. The events E (roll is even) and G (roll is greater than 4) are *not* disjoint—specifically, $E \text{ and } G = \{6\}$—so $P(E \text{ or } G) = P(E) + P(G) - P(E \text{ and } G) = \frac{3}{6} + \frac{2}{6} - \frac{1}{6} = \frac{4}{6} = \frac{2}{3}$.

4.96. Let A be the event "next card is an ace" and B be "two of Slim's four cards are aces." Then, $P(A \mid B) = \frac{2}{48}$ because (other than those in Slim's hand) there are 48 cards, of which 2 are aces.

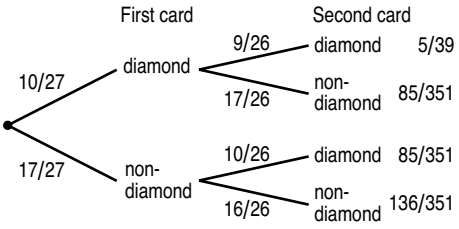
4.97. **(a)** There are 2100 grades in the Health and Human Services (HHS) row, so $P(\text{HHS}) = \frac{2100}{10{,}000} = 0.21$. **(b)** There are 3392 grades in the A column, so $P(\text{A}) = \frac{3392}{10{,}000} = 0.3392$. **(c)** There are 882 A's among the 2100 HHS grades, so $P(\text{A} \mid \text{HHS}) = \frac{882}{2100} = 0.42$. **(d)** The events "getting an A" and "being in the HHS school" are not independent; specifically, A's are more common in HHS than overall.

4.98. Let A_1 = "the next card is a diamond" and A_2 = "the second card is a diamond." We wish to find $P(A_1 \text{ and } A_2)$. We assume that the three diamonds in Slim's hand are the *only* diamonds visible. In that case, there are 27 unseen cards, of which 10 are diamonds, so $P(A_1) = \frac{10}{27}$, and $P(A_2 \mid A_1) = \frac{9}{26}$, so $P(A_1 \text{ and } A_2) = \frac{10}{27} \times \frac{9}{26} = \frac{5}{39} \doteq 0.1282$.

Note: *Technically, we wish to find $P(A_1 \text{ and } A_2 \mid B)$, where B is the given event (25 cards visible, with 3 diamonds in Slim's hand). Also, if there are an additional k diamonds visible aside from those in Slim's hand, this probability would be $\frac{10-k}{27} \times \frac{9-k}{26}$.*

4.99. Let B be the event "the grade is a B" and E be the event "the grade came from the School of Engineering and Physical Sciences (EPS). There are 1600 grades in EPS, of which 432 are B's, so $P(B \mid E) = \frac{432}{1600} = 0.27$. This agrees with the conditional probability found by the definition: $P(B \text{ and } E) = \frac{432}{10{,}000} = 0.0432$ and $P(E) = \frac{1600}{10{,}000} = 0.16$, so $P(B \mid E) = \frac{P(B \text{ and } E)}{P(E)} = \frac{0.0432}{0.16} = 0.27$.

4.100. The tree diagram shows the probability found in Exercise 4.98 on the top branch. The middle two branches (added together) give the probability that Slim gets exactly one diamond from the next two cards, and the bottom branch is the probability that neither card is a diamond.



4.101. Let B = "student is a binge drinker" and M = "student is male." **(a)** The four probabilities sum to $0.11 + 0.12 + 0.32 + 0.45 = 1$. **(b)** $P(B^c) = 0.32 + 0.45 = 0.77$. **(c)** $P(B^c \mid M) = \frac{P(B^c \text{ and } M)}{P(M)} = \frac{0.32}{0.11+0.32} \doteq 0.7442$. **(d)** In the language of this chapter, the events are not independent. An attempt to phrase this for someone who has not studied this material might say something like, "Knowing a student's gender gives some information about whether or not that student is a binge drinker."

Note: *Specifically, male students are slightly more likely to be binge drinkers. This statement might surprise students who look at the table and note that the proportion of binge drinkers in the men's columns is smaller than that proportion in the women's column. We cannot compare those proportions directly; we need to compare the* conditional *probabilities of binge drinkers within each given gender (see the solution to the next exercise.)*

4.102. Let B = "student is a binge drinker" and M = "student is male." **(a)** These two probabilities are given as entries in the table: $P(M \text{ and } B) = 0.11$ and $P(M^c \text{ and } B) = 0.12$. **(b)** These are conditional probabilities: $P(B \mid M) = \frac{P(B \text{ and } M)}{P(M)} = \frac{0.11}{0.11+0.32} \doteq 0.2558$ and $P(B \mid M^c) = \frac{P(B \text{ and } M^c)}{P(M^c)} = \frac{0.12}{0.12+0.45} \doteq 0.2105$. **(c)** The fact that $P(B \mid M) > P(B \mid M^c)$ indicates that male students are more likely to be binge drinkers (see the comment in the solution to the previous exercise). The other comparison, $P(M \text{ and } B) < P(M^c \text{ and } B)$, is more a reflection of the fact that the survey reported responses for more women (57%) than men (43%) and does not by itself allow for comparison of binge drinking between the genders. (To understand this better, imagine a more extreme case, where, say, 90% of respondents were women ....)

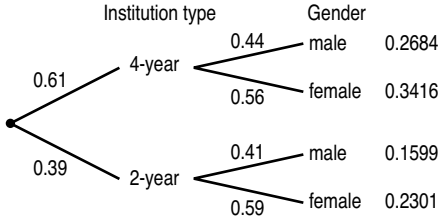
4.103. Let M = "male" and F = "attends a 4-year institution." We have been given $P(F) = 0.61$, $P(F^c) = 0.39$, $P(M \mid F) = 0.44$ and $P(M \mid F^c) = 0.41$. **(a)** To create the table, observe that:

	Men	Women
4-year	0.2684	0.3416
2-year	0.1599	0.2301

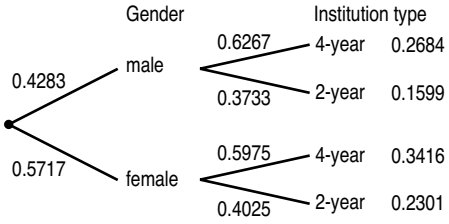
$$P(M \text{ and } F) = P(M \mid F)P(F) = (0.44)(0.61) = 0.2684$$

And similarly, $P(M \text{ and } F^c) = P(M \mid F^c)P(F^c) = (0.41)(0.39) = 0.1599$. The other two entries can be found in a similar fashion or by observing that, for example, the two numbers on the first row must sum to $P(F) = 0.61$.
(b) $P(F \mid M^c) = \frac{P(F \text{ and } M^c)}{P(M^c)} = \frac{0.3416}{0.3416+0.2301} \doteq 0.5975$.

4.104. The branches of this tree diagram have the probabilities given in Exercise 4.103, and the branches end with the probabilities found in the solution to that exercise.



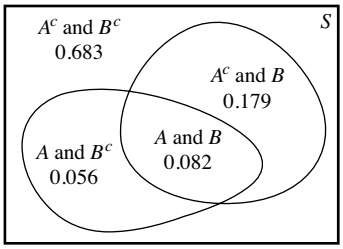
4.105. Let M = "male" and F = "attends a 4-year institution." For this tree diagram, we need to compute $P(M) = 0.2684 + 0.1599 = 0.4283$, $P(M^c) = 0.3416 + 0.2301 = 0.5717$, as well as $P(F \mid M)$, $P(F \mid M^c)$, $P(F^c \mid M)$, and $P(F^c \mid M^c)$. For example, $P(F \mid M) = \frac{P(F \text{ and } M)}{P(M)} = \frac{0.2684}{0.4283} \doteq 0.6267$. All the computations for this diagram are "inconvenient" because they require that we work *backward* from the ending probabilities, instead of working *forward* from the given probabilities (as we did in the previous tree diagram).



4.106. $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) = 0.138 + 0.261 - 0.082 = 0.317$.

4.107. $P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)} = \frac{0.082}{0.261} \doteq 0.3142$. If A and B were independent, then $P(A \mid B)$ would equal $P(A)$, and also $P(A \text{ and } B)$ would equal the product $P(A)P(B)$.

4.108. **(a)** $\{A \text{ and } B\}$: household is both prosperous and educated; $P(A \text{ and } B) = 0.082$ (given). **(b)** $\{A \text{ and } B^c\}$: household is prosperous but not educated; $P(A \text{ and } B^c) = P(A) - P(A \text{ and } B) = 0.056$. **(c)** $\{A^c \text{ and } B\}$: household is not prosperous but is educated; $P(A^c \text{ and } B) = P(B) - P(A \text{ and } B) = 0.179$. **(d)** $\{A^c \text{ and } B^c\}$: household is neither prosperous nor educated; $P(A^c \text{ and } B^c) = 0.683$ (so that the probabilities add to 1).



4.109. **(a)** "The vehicle is a light truck" $= A^c$; $P(A^c) = 0.69$. **(b)** "The vehicle is an imported car" $= A$ and B. To find this probability, note that we have been given $P(B^c) = 0.78$ and $P(A^c \text{ and } B^c) = 0.55$. From this we can determine that $78\% - 55\% = 23\%$ of vehicles sold were domestic cars—that is, $P(A \text{ and } B^c) = 0.23$—so $P(A \text{ and } B) = P(A) - P(A \text{ and } B^c) = 0.31 - 0.23 = 0.08$.

	$P(A) = 0.31$	$P(A^c) = \mathbf{0.69}$
$P(B) = 0.22$	$P(A \text{ and } B) = 0.08$	$P(A^c \text{ and } B) = 0.14$
$P(B^c) = \mathbf{0.78}$	$P(A \text{ and } B^c) = 0.23$	$P(A^c \text{ and } B^c) = \mathbf{0.55}$

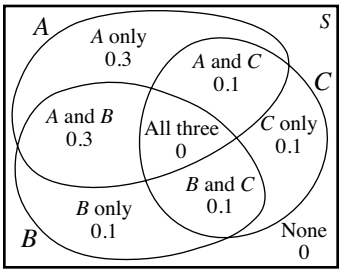
Note: *The table shown here summarizes all that we can determine from the given information* (**bold**).

4.110. Let A be the event "income $\geq$ \$1 million" and B be "income $\geq$ \$100,000." Then "A and B" is the same as A, so:

$$P(A \mid B) = \frac{P(A)}{P(B)} = \frac{\frac{240{,}128}{312{,}226{,}042}}{\frac{12{,}757{,}005}{312{,}226{,}042}} = \frac{240{,}128}{12{,}757{,}005} \doteq 0.01882$$

4.111. See also the solution to Exercise 4.109, especially the table of probabilities given there. **(a)** $P(A^c \mid B) = \frac{P(A^c \text{ and } B)}{P(B)} = \frac{0.14}{0.22} \doteq 0.6364$. **(b)** The events A^c and B are *not* independent; if they were, $P(A^c \mid B)$ would be the same as $P(A^c) = 0.69$.

4.112. To find the probabilities in this Venn diagram, begin with $P(A \text{ and } B \text{ and } C) = 0$ in the center of the diagram. Then each of the two-way intersections $P(A \text{ and } B)$, $P(A \text{ and } C)$, and $P(B \text{ and } C)$ go in the remainder of the overlapping areas; if $P(A \text{ and } B \text{ and } C)$ had been something other than 0, we would have subtracted this from each of the two-way intersection probabilities to find, for example, $P(A \text{ and } B \text{ and } C^c)$. Next, determine $P(A \text{ only})$ so that the total probability of the regions that make up the event A is 0.7. Finally, $P(\text{none}) = P(A^c \text{ and } B^c \text{ and } C^c) = 0$ because the total probability inside the three sets A, B, and C is 1.



4.113. We seek $P(\text{at least one offer}) = P(A \text{ or } B \text{ or } C)$; we can find this as $1 - P(\text{no offers}) = 1 - P(A^c \text{ and } B^c \text{ and } C^c)$. We see in the Venn diagram of Exercise 4.112 that this probability is 1.

4.114. This is $P(A \text{ and } B \text{ and } C^c)$. As was noted in Exercise 4.112, because $P(A \text{ and } B \text{ and } C) = 0$, this is the same as $P(A \text{ and } B) = 0.3$.

4.115. $P(B \mid C) = \dfrac{P(B \text{ and } C)}{P(C)} = \dfrac{0.1}{0.3} = \dfrac{1}{3}$. $P(C \mid B) = \dfrac{P(B \text{ and } C)}{P(B)} = \dfrac{0.1}{0.5} = 0.2$.

4.116. Let $W =$ "the degree was earned by a woman" and $P =$ "the degree was a professional degree." **(a)** To construct the table (below), divide each entry by the grand total of all entries (2403); for example, $\frac{933}{2403} \doteq 0.3883$ is the fraction of all degrees that were bachelor's degrees awarded to women. Some students may also find the row totals (1412 and 991) and the column totals (1594, 662, 95, 52) and divide those by the grand total; for example, $\frac{1594}{2403} \doteq 0.6633$ is the fraction of all degrees that were bachelor's degrees. **(b)** $P(W) = \frac{1412}{2403} \doteq 0.5876$ (this is one of the optional marginal probabilities from the table below). **(c)** $P(W \mid P) = \frac{51/2403}{95/2403} = \frac{51}{95} \doteq 0.5368$. (This is the "Female" entry from the "Professional" column, divided by that column's total.) **(d)** W and P are *not* independent; if they were, the two probabilities in (b) and (c) would be equal.

	Bachelor's	Master's	Professional	Doctorate	Total
Female	0.3883	0.1673	0.0212	0.0108	0.5876
Male	0.2751	0.1082	0.0183	0.0108	0.4124
Total	0.6633	0.2755	0.0395	0.0216	1.0000

4.117. Let M be the event "the person is a man" and B be "the person earned a bachelor's degree." **(a)** $P(M) = \frac{991}{2403} \doteq 0.4124$. Or take the answer from part (b) of the previous exercise and subtract from 1. **(b)** $P(B \mid M) = \frac{661/2403}{991/2403} = \frac{661}{991} \doteq 0.6670$. (This is the "Bachelor's" entry from the "Male" row, divided by that row's total.) **(c)** $P(M \text{ and } B) = P(M)\,P(B \mid M) \doteq (0.4124)(0.6670) \doteq 0.2751$. This agrees with the directly computed probability: $P(M \text{ and } B) = \frac{661}{2403} \doteq 0.2751$.

4.118. Each unemployment rate is computed as shown on the right. (Alternatively, subtract the number employed from the number in the labor force, then divide that difference by the number in the labor force.) Because these rates (probabilities) are different, education level and being employed are not independent.

Did not finish HS	$1 - \frac{11,552}{12,623} \doteq 0.0848$
HS/no college	$1 - \frac{36,249}{38,210} \doteq 0.0513$
Some college	$1 - \frac{32,429}{33,928} \doteq 0.0442$
College graduate	$1 - \frac{39,250}{40,414} \doteq 0.0288$

4.119. **(a)** Add up the numbers in the first and second columns. We find that there are 186,210 thousand (i.e., over 186 million) people aged 25 or older, of which 125,175 thousand are in the labor force, so $P(L) = \frac{125,175}{186,210} \doteq 0.6722$. **(b)** $P(L \mid C) = \frac{P(L \text{ and } C)}{P(C)} = \frac{40,414}{51,568} \doteq 0.7837$. **(c)** L and C are *not* independent; if they were, the two probabilities in (a) and (b) would be equal.

4.120. For the first probability, add up the numbers in the third column. We find that there are 119,480 thousand (i.e., over 119 million) employed people aged 25 or older. Therefore, $P(C \mid E) = \frac{P(C \text{ and } E)}{P(E)} = \frac{39,250}{119,480} \doteq 0.3285$.

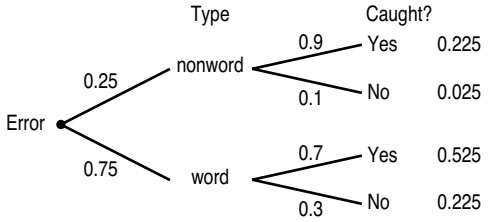
For the second probability, we use the total number of college graduates in the population: $P(E \mid C) = \frac{P(C \text{ and } E)}{P(C)} = \frac{39,250}{51,568} \doteq 0.7611$.

4.121. The population includes retired people who have left the labor force. Retired persons are more likely than other adults to have not completed high school; consequently, a relatively large number of retired persons fall in the "did not finish high school" category.

Note: *Details of this lurking variable can be found in the Current Population Survey annual report on "Educational Attainment in the United States." For 2006, this report says that among the 65-and-over population, about 24.8% have not completed high school, compared to about 19.3% of the under-65 group.*

4.122. The given probabilities are $P(W) = 0.62$, $P(E) = 0.17$, and $P(W \mid E) = 0.8$. By the multiplication rule, $P(E \text{ and } W) = P(E)\,P(W \mid E) = (0.17)(0.8) = 0.136$. Therefore, $P(E \mid W) = \frac{P(E \text{ and } W)}{P(W)} = \frac{0.136}{0.62} \doteq 0.2194$.

4.123. Tree diagram at right. The numbers on the right side of the tree are found by the multiplication rule; for example, P("nonword error" and "caught") $= P(N \text{ and } C) = P(N)\,P(C \mid N) = (0.25)(0.9) = 0.225$. A proofreader should catch about $0.225 + 0.525 = 0.75 = 75\%$ of all errors.



4.124. With B, M, and D representing the three kinds of degrees, and W meaning the degree recipient was a woman, we have been given:

$$P(B) = 0.73, \quad P(M) = 0.21, \quad P(D) = 0.06,$$
$$P(W \mid B) = 0.48, \quad P(W \mid M) = 0.42, \quad P(W \mid D) = 0.29$$

Therefore, we find $P(W) = P(W \text{ and } B) + P(W \text{ and } M) + P(W \text{ and } D) = P(B)\,P(W \mid B) + P(M)\,P(W \mid M) + P(D)\,P(W \mid D) = 0.456$, so:

$$P(B \mid W) = \frac{P(B \text{ and } W)}{P(W)} = \frac{P(B)\,P(W \mid B)}{P(W)} = \frac{0.3504}{0.456} \doteq 0.7684$$

4.125. **(a)** Her brother has type aa, and he got one allele from each parent. But neither parent is albino, so neither could be type aa. **(b)** The table on the right shows the possible combinations, each of which is equally likely, so $P(aa) = 0.25$, $P(Aa) = 0.5$, and $P(AA) = 0.25$. **(c)** Beth is either AA or Aa, and $P(AA \mid \text{not } aa) = \frac{0.25}{0.75} = \frac{1}{3}$, while $P(Aa \mid \text{not } aa) = \frac{0.50}{0.75} = \frac{2}{3}$.

	A	a
A	AA	Aa
a	Aa	aa

4.126. **(a)** If Beth is Aa, then the first table on the right gives the (equally likely) allele combinations for a child, so $P(\text{child is non-albino} \mid \text{Beth is } Aa) = \frac{1}{2}$. If Beth is AA, then as the second table shows, their child will definitely be type Aa (and non-albino), so $P(\text{child is non-albino} \mid \text{Beth is } AA) = 1$. **(b)** We have:

	A	a
a	Aa	aa
a	Aa	aa

	A	A
a	Aa	Aa
a	Aa	Aa

$$\begin{aligned} P(\text{child is non-albino}) &= P(\text{child } Aa \text{ and Beth } Aa) + P(\text{child } Aa \text{ and Beth } AA) \\ &= P(\text{Beth } Aa)\,P(\text{child } Aa \mid \text{Beth } Aa) + P(\text{Beth } AA)\,P(\text{child } Aa \mid \text{Beth } AA) \\ &= \tfrac{2}{3} \cdot \tfrac{1}{2} + \tfrac{1}{3} \cdot 1 = \tfrac{2}{3} \end{aligned}$$

Therefore, $P(\text{Beth is } Aa \mid \text{child is } Aa) = \frac{1/3}{2/3} = \frac{1}{2}$.

4.127. Let T be the event "test is positive" and C be the event "Jason is a carrier." Since the given information says that the test is never positive for noncarriers, it clearly must be the case that $P(C \mid T) = 1$.

To confirm this, note that (if there is no human error) we have $P(T \text{ and } C^c) = 0$ and $P(T) = P(T \text{ and } C) + P(T \text{ and } C^c) = P(T \text{ and } C) = P(T)\,P(T \mid C) = (0.04)(0.9) = 0.036$. Therefore, $P(C \mid T) = \frac{P(C \text{ and } T)}{P(T)} = \frac{0.036}{0.036} = 1$.

4.128. Let C be the event that Julianne is a carrier, and let D be the event that Jason's and Julianne's child has the disease. We have been given $P(C) = \frac{2}{3}$, $P(D \mid C) = \frac{1}{4}$, and $P(D \mid C^c) = 0$. Therefore, $P(D^c) = P(C)\,P(D^c \mid C) + P(C^c)\,P(D^c \mid C^c) = \left(\frac{2}{3}\right)\left(\frac{3}{4}\right) + \left(\frac{1}{3}\right)(1) = \frac{1}{2} + \frac{1}{3} = \frac{5}{6}$, and $P(C \mid D^c) = \frac{1/2}{5/6} = \frac{3}{5}$.

4.129. Let C be the event "Toni is a carrier," T be the event "Toni tests positive," and D be "her son has DMD." We have $P(C) = \frac{2}{3}$, $P(T \mid C) = 0.7$, and $P(T \mid C^c) = 0.1$. Therefore, $P(T) = P(T \text{ and } C) + P(T \text{ and } C^c) = P(C)\,P(T \mid C) + P(C^c)\,P(T \mid C^c) = \left(\frac{2}{3}\right)(0.7) + \left(\frac{1}{3}\right)(0.1) = 0.5$, and:

$$P(C \mid T) = \frac{P(T \text{ and } C)}{P(C)} = \frac{(2/3)(0.7)}{0.5} = \frac{14}{15} \doteq 0.9333$$

4.130. **(a)** A and B are disjoint. (If A happens, B did not.) **(b)** A and B are independent. (A concerns the first roll, B the second.) **(c)** A and B are independent. (A concerns the second roll, B the first.) **(d)** A and B are neither disjoint nor independent. (If A happens, then so does B.)

4.131. **(a)** $P(A) = \frac{6}{36} = \frac{1}{6}$ and $P(B) = \frac{15}{36} = \frac{5}{12}$. **(b)** $P(A) = \frac{6}{36} = \frac{1}{6}$ and $P(B) = \frac{15}{36} = \frac{5}{12}$. **(c)** $P(A) = \frac{15}{36} = \frac{5}{12}$ and $P(B) = \frac{10}{36} = \frac{5}{18}$. **(d)** $P(A) = \frac{15}{36} = \frac{5}{12}$ and $P(B) = \frac{10}{36} = \frac{5}{18}$.

4.132. **(a)** The mean is $\mu_X = (1)(0.2) + (2)(0.6) + (3)(0.2) = 2$. The variance is $\sigma_X^2 = (1-2)^2(0.2) + (2-2)^2(0.6) + (3-2)^2(0.2) = 0.4$, so the standard deviation is $\sigma_X = \sqrt{0.4} \doteq 0.6325$. **(b)** & **(c)** To achieve a mean of 2 with possible values 1, 2, and 3, the distribution must be symmetric; that is, the probability at 1 must equal the probability at 3 (so that 2 would be the balance point of the distribution). Let p be the probability assigned to 1 (and also to 3) in the new distribution. A larger standard deviation is achieved when $p > 0.2$, and a smaller standard deviation arises when $p < 0.2$. In either case, the new standard deviation is $\sqrt{2p}$.

Value of X	1	2	3
Probability	0.2	0.6	0.2
(b) & (c)	p	$1-2p$	p

4.133. For each bet, the mean is the winning probability times the winning payout. (A loss has a payout of \$0, so it contributes nothing to the mean.) Each mean payoff is $\$10 = (\$20)(0.5) = (\$15)\left(\frac{2}{3}\right) = (\$12)\left(\frac{5}{6}\right)$; for a \$10 bet, the net gain is \$0.

4.134. $P(\text{A}) = P(\text{B}) = \cdots = P(\text{F}) = \frac{0.72}{6} = 0.12$ and $P(1) = \cdots = P(8) = \frac{1-0.72}{8} = 0.035$.

4.135. (a) All probabilities are greater than or equal to 0, and their sum is 1. **(b)** Let R_1 be Taster 1's rating and R_2 be Taster 2's rating. Add the probabilities on the diagonal (upper left to lower right): $P(R_1 = R_2) = 0.03 + 0.07 + 0.25 + 0.20 + 0.06 = 0.61$. **(c)** $P(R_1 > 3) = 0.39$ (the sum of the ten numbers in the bottom two rows), and $P(R_2 > 3) = 0.39$ (the sum of the ten numbers in the right two rows).

4.136. (a) The mean is $\mu_X = (1)(0.4) + (1.5)(0.2) + (2)(0.2) + (4)(0.1) + (10)(0.1) = 2.5$ million dollars. The variance is:

$$\begin{aligned}\sigma_X^2 &= (1-2.5)^2(0.4) + (1.5-2.5)^2(0.2) + (2-2.5)^2(0.2) \\ &\quad + (4-2.5)^2(0.1) + (10-2.5)^2(0.1) \\ &= 7\end{aligned}$$

The standard deviation is $\sigma_X = \sqrt{7} \doteq 2.6458$ million dollars. **(b)** $\mu_Y = \mu_{0.9X-0.2} = 0.9\mu_X - 0.2 = 2.05$ million dollars, and $\sigma_Y = \sigma_{0.9X-0.2} = 0.9\sigma_X \doteq 2.3812$ million dollars.

4.137. (a) The probability of winning nothing is $1 - \left(\frac{1}{10{,}000} + \frac{1}{1{,}000} + \frac{1}{100} + \frac{1}{20}\right) = 0.9389$. **(b)** The mean is $\mu = (\$1000)\left(\frac{1}{10{,}000}\right) + (\$250)\left(\frac{1}{1{,}000}\right) + (\$100)\left(\frac{1}{100}\right) + (\$10)\left(\frac{1}{20}\right) = \$1.85$. **(c)** $\sigma^2 = (\$998.15)^2\left(\frac{1}{10{,}000}\right) + (\$248.15)^2\left(\frac{1}{1{,}000}\right) + (\$98.15)^2\left(\frac{1}{100}\right) + (\$8.15)^2\left(\frac{1}{20}\right) \doteq 260.86$, so $\sigma \doteq \$16.1513$.

4.138. As $\sigma_{a+bX} = b\sigma_X$ and $\sigma_{c+dY} = d\sigma_Y$, we need $b = \frac{100}{106}$ and $d = \frac{100}{109}$. With these choices for b and d, we have $\mu_{a+bX} = a + b\mu_X \doteq a + 419.8113$, so $a \doteq 80.1887$, and $\mu_{c+dX} = c + d\mu_Y \doteq c + 519.2661$, so $c \doteq -19.2661$.

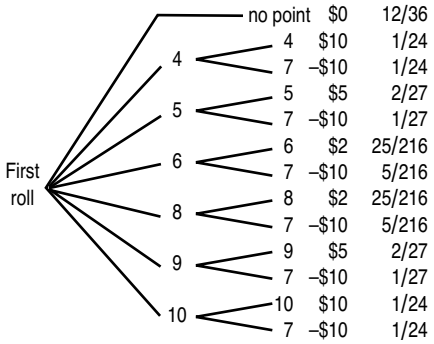
4.139. This is the probability of 19 (independent) losses, followed by a win; by the multiplication rule, this is $0.994^{19} \cdot 0.006 \doteq 0.005352$.

4.140. (a) $P(\text{win the jackpot}) = \left(\frac{1}{20}\right)\left(\frac{8}{20}\right)\left(\frac{1}{20}\right) = 0.001$. **(b)** The other symbol can show up on the middle wheel, with probability $\left(\frac{1}{20}\right)\left(\frac{12}{20}\right)\left(\frac{1}{20}\right) = 0.0015$, or on either of the outside wheels, with probability $\left(\frac{19}{20}\right)\left(\frac{8}{20}\right)\left(\frac{1}{20}\right) = 0.019$. Therefore, combining all three cases, we have $P(\text{exactly two bells}) = 0.0015 + 2 \cdot 0.019 = 0.053$.

4.141. The table shows conditional distributions given T (public or private institution) and given Y (2-year or 4-year institution). The four numbers in the "Given T" group are found by dividing each number in the table by the *column* total (so each column sums to 1). In the "Given Y" group, we divide each number by the *row* total (so each row sums to 1). For example: $P(\text{2-year} \mid \text{Public}) = \frac{639}{639+1061} \doteq 0.3759$, $P(\text{2-year} \mid \text{Private}) = \frac{1894}{1894+622} \doteq 0.7528$, and $P(\text{Public} \mid \text{2-year}) = \frac{639}{639+1894} \doteq 0.2523$.

	Given T		Given Y	
	Public	Private	Public	Private
2-year	0.3759	0.7528	0.2523	0.7477
4-year	0.6241	0.2472	0.6304	0.3696

4.142. In Exercise 4.133, the probabilities of winning each odds bet were given as $\frac{1}{2}$ for 4 and 10, $\frac{2}{3}$ for 5 and 9, and $\frac{5}{6}$ for 6 and 8. This tree diagram can get a bit large (and crowded). In the diagram shown on the right, the probabilities are omitted from the individual branches. The probability of winning an odds bet on 4 (with a net payout of \$10), for example, is $\left(\frac{3}{36}\right)\left(\frac{1}{2}\right) = \frac{1}{24}$; losing an odds bet on 4 has the same probability, and costs \$10 (the amount of the bet). Similarly, the probability of winning an odds bet on 5 is $\left(\frac{4}{36}\right)\left(\frac{2}{3}\right) = \frac{2}{27}$, and the probability of losing that bet is $\left(\frac{4}{36}\right)\left(\frac{1}{3}\right) = \frac{1}{27}$.



To confirm that this game is fair, one can multiply each payoff by its probability then add up all of those products. More directly, because each individual odds bet is fair (as was shown in the solution to Exercise 4.133), one can argue that taking the odds bet whenever it is available must be fair.

4.143. Student findings will depend on how much they explore the Web site. Individual growth charts include weight-for-age, height-for-age, weight-for-length, head circumference-for-age, and body mass index-for-age.

4.144. Let R_1 be Taster 1's rating and R_2 be Taster 2's rating. $P(R_1 = 3) = 0.01 + 0.05 + 0.25 + 0.05 + 0.01 = 0.37$, so:

$$P(R_2 > 3 \mid R_1 = 3) = \frac{P(R_2 > 3 \text{ and } R_1 = 3)}{P(R_1 = 3)} = \frac{0.05 + 0.01}{0.37} \doteq 0.1622$$

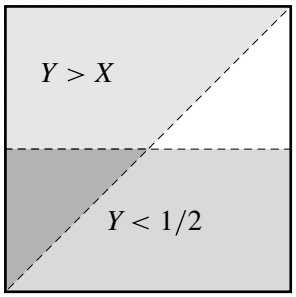
4.145. Let F be "adult is a full-time student," P be "adult is a part-time student," N be "adult is not a student," and A be "adult accesses Internet from someplace other than work or home."

We were given $P(F) = 0.041$ and $P(P) = 0.029$ so that $P(N) = 1 - 0.041 - 0.029 = 0.93$. Also, $P(A \mid F) = 0.58$, $P(A \mid P) = 0.30$, and $P(A \mid N) = 0.21$. Therefore, $P(A) = P(F)\,P(A \mid F) + P(P)\,P(A \mid P) + P(N)\,P(A \mid N) \doteq 0.22778$—about 22.8%.

4.146. Note first that $P(A) = \frac{1}{2}$ and $P(B) = \frac{2}{4} = \frac{1}{2}$. Now $P(B \text{ and } A) = P(\text{both coins are heads}) = 0.25$, so $P(B \mid A) = \frac{P(B \text{ and } A)}{P(A)} = \frac{0.25}{0.5} = 0.5 = P(B)$.

4.147. The event $\{Y < 1/2\}$ is the bottom half of the square, while $\{Y > X\}$ is the upper left triangle of the square. They overlap in a triangle with area 1/8, so:

$$P(Y < \tfrac{1}{2} \mid Y > X) = \frac{P(Y < \frac{1}{2} \text{ and } Y > X)}{P(Y > X)} = \frac{1/8}{1/2} = \frac{1}{4}$$



4.148. The response will be "no" with probability $0.35 = (0.5)(0.7)$. If the probability of plagiarism were 0.2, then $P(\text{student answers "no"}) = 0.4 = (0.5)(0.8)$. If 39% of students surveyed answered "no," then we estimate that $2 \cdot 39\% = 78\%$ have *not* plagiarized, so about 22% have plagiarized.

